Reflective twisted nematic liquid crystal displays. II. Elimination of retardation film and rear polarizer


Department of Electrical and Electronic Engineering, Center for Display Research, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong

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Reflective twisted nematic liquid crystal displays consisting of just one input polarizer, a liquid crystal cell, and a rear reflector are discussed. We first show that all such display modes reported in the literature can be depicted systematically in a series of parameter space diagrams. Then by making use of such diagrams as a guide, we show that other new high quality reflective display modes can be obtained. The three parameters that are varied are the input polarizer angle \( \alpha \), the liquid crystal twist angle \( \phi \), and the cell gap–liquid crystal birefringence product \( d\Delta n \). A full numerical simulation which takes into account of the liquid crystal director deformation under the application of an electric field was used to optimize such displays. Experimental results agree well with the theoretical predictions. © 1997 American Institute of Physics. [S0021-8979(97)02923-X]

I. INTRODUCTION

Ordinary twisted nematic (TN) and supertwisted nematic (STN) liquid crystal displays (LCD) always have a front and a rear polarizer and operate in the transmissive or transflective modes. In the transflective mode, a mirror is placed in the back so that light traverses the LC cell twice. When the rear polarizer is eliminated, the display will be in the true reflective mode. The brightness of such reflective displays is expected to increase considerably because of less absorption and fewer scattering losses. In addition, there are several other advantages such as the elimination of parallax and viewing shadows, higher pixel density, and simpler manufacturing procedures. They can also be applied to crystalline silicon based active matrix LCD which can only operate in the reflective mode.

Many types of reflective LCDs have been proposed. Broadly speaking, they can be divided into two categories: those that rely on the twisted nematic effect, and those that do not. The latter type of displays do not require any polarizers. Some examples of such displays are the bistable cholesteric display, and the guest–host TN display. In this article, we shall be concerned with nematic type displays.

In the literature, several reflective nematic LCD modes have been proposed and demonstrated. They include (1) the hybrid field effect (HFE) mode, \(^7\) \(^8\) (2) the twisted nematic-electrically controlled birefringence (TN-ECB) mode, \(^9\) \(^14\) (3) the mixed twisted nematic (MTN) mode, \(^15\) and (4) the self-compensated twisted nematic (SCTN) mode. \(^16\) In this article, we shall propose and demonstrate a new variation, the reflective TN (RTN) mode, \(^17\) which has the merits of relatively large cell gaps, high light efficiency, low color dispersion, and high contrast. In this article, we shall explain the similarities and differences between all of these modes using the parameter space diagram. \(^18\) Basically the MTN, TN-ECB, and SCTN modes are quite similar in that, like ordinary TN displays, the homeotropic state is used either as the bright or dark state. The HFE and RTN belong to another class of displays where both the bright and dark states have finite voltages and are nonhomeotropic.

In the following, we shall first introduce an improved formulation of the parameter space diagram in Sec. II. We shall use such diagrams to explain the operation of various reflective nematic LCD modes in Sec. III. The design of the RTN will then be discussed in Sec. IV. Section V presents the experimental results, comparing the theoretical simulation with some real devices.

It should be emphasized that all the display modes to be discussed in this article have twist angles \( \approx 90^\circ \). Thus they have shallow reflectance-voltage curves (RVCs) and are therefore not suitable for multiplexing. They have applications mostly in active matrix driven displays. For example, the HFE has been applied in optically addressed liquid crystal light valves (LCLVs). \(^7\) \(^8\) The MTN has been used in an \( \alpha \)-Si active matrix LCD. \(^19\) All of these reflective nematic modes, particularly the RTN mode, can be used as reflective projection light valves or in direct view miniature displays using crystalline silicon as the backplane. \(^2\)

II. FORMULATION OF THE PARAMETER SPACE

The starting point of all of our discussions of LCD operating modes is the parameter space. \(^18\) The idea is that the static optical properties of any reflective nematic LC cell is determined mainly by three parameters: the angle between the input director and the polarizer \( \alpha \), the liquid crystal twist angle \( \phi \), and the thickness-birefringence product \( d\Delta n \) of the LC cell. Thus the reflectance of the reflective nematic display can be plotted in a two-dimensional (2D) contour dia-

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\(^{a}\)Author to whom correspondence should be addressed. Electronic mail: eekwok@usthk.ust.hk
gram with one of the parameters fixed. For normal incidence, the reflectance can be calculated using the simple Jones matrix. The Jones matrix for the LC cell with uniform twist and zero tilt is given by 18,20–22
\[
M = \begin{pmatrix} A - iB & -C - iD \\ C - iD & A + iB \end{pmatrix},
\]
(1)
where
\[
A = \cos \phi \cos \beta d + \frac{q}{\beta} \sin \phi \sin \beta d, \tag{2}
\]
\[
B = \frac{k_a}{\beta} \cos \phi \sin \beta, \tag{3}
\]
\[
C = \sin \phi \cos \beta d - \frac{q}{\beta} \cos \phi \sin \beta d, \tag{4}
\]
\[
D = \frac{k_a}{\beta} \sin \phi \sin \beta d. \tag{5}
\]
In Eqs. (2)–(5), \(q = 2\pi/p\), where \(p\) is the pitch of the LC cell, and
\[
\beta = (k_a^2 + q^2)^{1/2}. \tag{6}
\]
For a twisted nematic cell, the pitch is related to \(\phi\) by
\[
qd = \phi. \tag{7}
\]
Also
\[
k_a = \frac{\pi \Delta n}{\lambda}, \tag{8}
\]
where
\[
\Delta n = n_e(\theta) - n_o\tag{9}
\]
is the birefringence of the tilted liquid crystals. Finally \(n_e(\theta)\) is the extraordinary index at an average director tilt angle of \(\theta\) and is given by the usual expression
\[
\frac{1}{n_e^2(\theta)} = \frac{\cos^2 \theta}{n_o^2} + \frac{\sin^2 \theta}{n_e^2}. \tag{10}
\]
Notice that even though there are three variables in the Jones matrix, only two parameters are needed to completely define \(M\), since \(d\) and \(\Delta n\) always appear together as \(d\Delta n\).

Figures 1 and 2 show two possible viewing geometries for the reflective display. Figure 1 defines the coordinate system used in our calculations. The input director of the LC cell is assumed to be along the \(x\) axis. The incident light first passes through a polarizer with an angle \(\alpha\) to the \(x\) axis. The Jones matrix of the LC cell upon reflection can be related to \(M\) by a rotation of the coordinate system and a change in the twist direction of the LC cell. Hence the reflectance of the reflective nematic display is given by
\[
R = \left| (-\sin \alpha \cos \alpha) \cdot H \cdot M(\phi) \cdot H^{-1} \cdot M(-\phi) \cdot \begin{pmatrix} \cos \alpha \\ \sin \alpha \end{pmatrix} \right|^2, \tag{11}
\]
where \(H\) is the rotation matrix
\[
H = \begin{pmatrix} \cos \phi \sin \phi \\ -\sin \phi \cos \phi \end{pmatrix}. \tag{12}
\]
Notice that the transformation matrix \(H\) is different from that in Ref. 18 because of the different handedness assumed. In this formulation, it is clearly seen that the RTN behaves like a double layer supertwist nematic display (D-STN)28 except that the directors in the matching layer are not at right angles but are parallel. So the RTN has some self-compensation properties.

Figures 3(a)–3(d) show the reflectance of a reflective nematic display as a function of \(d\Delta n\) and \(\phi\) for \(\alpha\) ranging from \(0^\circ\) to \(45^\circ\). The case of \(\alpha = 0^\circ\) has been discussed before.18 The extrema in reflectance, which show up as wells in Fig. 3, are called the TN-ECB modes.9,10 with \((\phi, d\Delta n)\) values given by
\[
\phi = (2N - 1) \frac{\pi}{2\sqrt{2}} \tag{13}
\]
and
\[
d\Delta n = (2N - 1) \frac{\lambda}{2\sqrt{2}}, \quad \text{for } N = 1, 2, 3, \ldots \tag{14}
\]
The extrema in reflectance are \(0\%) if a \(\|\|\) polarizer geometry is used (e.g., a sheet polarizer as shown in Fig. 1). If a prism
beam splitter type polarizer is used, as shown in Fig. 2, then in viewing from the orthogonal direction, the extrema in Fig. 3 correspond to \( R = 100\% \). This case will be referred to as the cross polarizer \(^{i}\!-\!^{i}\!\) geometry. In the discussions to follow, a \(||-\|\) polarizer geometry is generally assumed implicitly. In practice, the \( ||-\| \) polarizer geometry is applicable to direct view displays and the \( ||-\| \) geometry is appropriate for projection displays.

In Fig. 3, we plot the parameter space for both positive and negative values of \( \phi \). The positive and negative TN-ECB modes are the same for \( \alpha = 0^\circ \). However, it can be seen in Figs. 3(b)–3(d) that the parameter space is asymmetrical in \( \phi \) if \( \alpha \) is nonzero. As a matter of fact, the TN-ECB wells creep systematically towards the center line of \( \phi = 0 \), and interlace each other, as \( \alpha \) varies from \( 0^\circ \) to \( 45^\circ \). As \( \alpha \) increases further, these wells separate again. The \( \alpha = 90^\circ \) parameter space is identical to the \( \alpha = 0^\circ \) parameter space. At \( \alpha = 45^\circ \), the modes along \( \phi = 0 \) are exactly the normal ECB modes.

The following symmetry conditions can be observed for the parameter space: (1) at \( \alpha = 0^\circ \) or \( 90^\circ \), + and - modes of the same order are degenerate. (2) \( \alpha \) and \( \alpha \pm 90^\circ \) yields the same parameter space. For example, \( \alpha = -60^\circ \), \( 30^\circ \), \( 120^\circ \), and \( 210^\circ \) are equivalent. (3) The parameter spaces for \( \alpha \) and \( 90^\circ - \alpha \) are the same if \( \phi \rightarrow -\phi \). These symmetry conditions are very useful in eliminating confusion about various polarizer conditions and the sign of the twist angle.

**III. COMPARISON OF DIFFERENT REFLECTIVE LCD MODES**

We shall examine the various reflective nematic modes discussed in the literature using Fig. 3. As mentioned above, we shall only consider reflective nematic LCDs consisting of just an input polarizer, a liquid crystal cell, and a rear reflector. Historically, such a configuration has been used in reflective LCLVs in the form of the HFE mode. In this mode, the twisted angles are either \( 45^\circ \) (Ref. 7) or \( 90^\circ \). The angle \( \alpha \) is always \( 0^\circ \) or \( 90^\circ \). Its operation relies on the nonuniform twist of the LCD at finite voltages so that the output light is elliptically polarized. In a sense the HFE mode is somewhat between the true ECB mode and the waveguiding TN mode and is indeed a hybrid of both. The maximum reflectance is always less than 100\% since at these twist angles, the LCD is not truly optimized and the output is not linearly polarized.

Sonehara et al.\(^9\!,^{10}\) proposed the TN-ECB modes with \( \phi \) values of \( 63.6^\circ \) and \( 190.8^\circ \) without much analysis. The reflectance of such displays should in principle be 100\%. Lu \textit{et al.}\(^11\!-\!^{13}\) and Konovalov \textit{et al.}\(^14\) also obtained similar results by varying \( \phi \). Basically, in these modes, the output light

![FIG. 3. Parameter spaces for a ||-|| reflective LCD. (a) \( \alpha = 0^\circ \), (b) \( \alpha = 15^\circ \), (c) \( \alpha = 30^\circ \), and (d) \( \alpha = 45^\circ \). The constant reflectance contours are in steps of 10%. The middle of the wells corresponds to zero reflectance.](image)
becomes circularly polarized after going through the LC cell once. Therefore upon reflection and going through the LC cell again, the light becomes linearly polarized again but rotated by 90° relative to the input. We showed analytically that for \( \alpha = 0° \) or 90°, the optimal twist angles for TN-ECB cells were given by Eqs. (13) and (14).\(^{18}\)

More recently, Wu and Wu\(^{15}\) introduced the MTN mode and Yang proposed the SCTN mode.\(^{16}\) The MTN modes are shown to have excellent optical properties including low chromatic dispersion and extremely high contrast. The SCTN mode has the advantage of low operating voltages. These modes, as well as the HFE and TN-ECB modes can all be depicted in the parameter space diagrams shown in Fig. 3. In Fig. 3, we also indicate the approximate operating condition of the RTN mode that we shall elaborate on further in Sec. IV.

An important observation is that the static operating points of these reflective LCD modes are either inside the TN-ECB wells or outside these wells. The MTN, TN-ECB, and SCTN modes lie in the center of one of the wells and are normally black (NB). (Actually, the 90° MTN\(^{15}\) is slightly off the center of the well.) They can be called the “in-well” modes. The RTN and HFE modes are normally white (NW) and are located in between the TN-ECB wells in the parameter space diagrams. They can therefore be classified as the “out-well” modes. The operational behavior of these two types of displays is very different when a voltage is applied. Figure 4 shows the RVC of the in-well, NB mode, and the out-well, NW displays (||-|| polarizer geometry assumed). The upper curve is that calculated for the TN-ECB first minimum, while the lower curve is that calculated for the HFE mode. Details of the calculation will be shown in Sec. IV. The point here is that, when a voltage is applied to the cell, \( d\Delta n \) will approach zero as the LC becomes homeotropic. For the TN-ECB, MTN, and SCTN modes, the reflectance goes up monotonically as in the upper curve of Fig. 4. In the ||-|| polarizer geometry as assumed, the dark state corresponds to \( V=0 \), and the bright state corresponds to homeotropic alignment. Hence the bright state should have very little chromatic dispersion. This is the major advantage of these modes of operation.

On the other hand, for the HFE and RTN displays, the dark state is obtained only at a finite intermediate voltage. The “dark” state will not be necessarily dark either. Hence the contrast of the display depends on how small one can make the minimum reflectance of the RTN display to be for all wavelengths. The homeotropic state is never involved in the operation of these “out-well” displays, as seen from Fig. 4. The major contribution of this article is therefore in optimizing the RTN to obtain a good dark state with \( R=0 \) for all colors. It is obvious that chromatic dispersion is a major issue in obtaining a good reflective LCD as well. One advantage of the RTN type of reflective mode is that both the dark and bright operating voltages are now relatively low compared to the TN-ECB, MTN, and SCTN modes where a high voltage induced homeotropic alignment is required. In the RTN, the operating voltages are always smaller than the homeotropic voltage.

Note also that the designations NW and NB can be interchanged by using a ||-|| polarizer geometry, or by inserting a quarter-wave plate inside the display.\(^{19}\) In that case, \( R \) becomes \((1-R)\). Hence the RVCs in Fig. 4 will be flipped upside down with the dark state becoming the bright state and vice versa. However, for many applications, such ploys are not desirable or possible. For example, in a direct view active matrix backplane reflective display, the MTN mode has to work in conjunction with a quarter-wave retardation plate.\(^{19}\) The RTN will eliminate the need for the retardation plate. But for projection displays, it is possible to use a prism cube polarizing beam splitter as shown in Fig. 3 to achieve the ||-|| polarizer geometry. In this case, the contrast will always be good since the dark state corresponds to \( V=0 \) which can always be made very dark. In fact, the HFE mode has been operated in this manner quite successfully as a LCLV.\(^{7,8}\)

A few general comments can be made here about the optimization of the RTN mode. First, the parameter space diagrams of Fig. 3 should be used as an initial guide only. They are obtained for \( V=0 \). Since there is no distortion of the LC director until the Frederiks transition, these diagrams can show the operation of the display at the nonselect voltage only. However, at higher voltages, the LC director will no longer be uniform and the Jones matrix of Eqs. (1)–(5) will no longer be valid. A full dynamical simulation will be needed.\(^{24–26}\) Second, the lower order TN-ECB modes generally have less chromatic dispersion. In particular, the first TN-ECB minimum at \( \phi=63.6° \) should be utilized as much as possible. Actually, both the MTN and SCTN modes are variations of this first TN-ECB mode as shown in Figs. 3(b) and 3(c). One major problem with these modes, however, is

![Figure 4](image-url) Reflectance as a function of voltage for the TN-ECB and HFE modes for a ||-|| geometry. Other “in-well” modes are similar to the TN-ECB curve and “out-well” modes are similar to the HFE curve.
that the value of $d\Delta n$ is too small to be practical. For the TN-ECB mode as proposed originally by Sonehara and Okumura, $d\Delta n = 0.19 \, \mu m$. For example, if $\Delta n = 0.1$, the cell gap has to be $1.9 \, \mu m$ which is difficult to achieve reliably. For this reason, experimental results for the TN-ECB modes were far from ideal. One major advantage of the RTN mode is that the value of $d\Delta n$ is difficult to achieve reliably. For the optimized RTN, the RVC goes through a minimum reflection of nearly 0%. Hence the contrast will be extremely high for this particular wavelength. So for a NW display, it makes a big change is small. These points are actually quite easy to identify since they always correspond to a broadband peak. Figure 5 is an example of a $\alpha - d\Delta n$ parameter space for $\phi = 52^\circ$ at the voltage-off state. For example, in Fig. 5, the point ($-10^\circ$, 0.55 $\mu m$) will be a good candidate for further calculation.

The next step is to obtain the RVCs for these potential operating points. In order to obtain the RVC, we followed the standard procedure for LC modeling: First the one-dimensional Euler–Lagrange equations for director deformation were solved to give the director angles $\phi(z)$ and $\theta(z)$ for all values of $z$ inside the LC cell for any value of applied voltage. Then the reflectance was calculated by dividing the cell into many layers and treating each layer as a birefringent plate, and multiplying together all the Jones matrices.

Figure 6 shows the results of such a calculation. It shows the RVCs for the HFE mode, and the new optimized RTN. The values of the elastic constants used for this calculation are those of a typical liquid crystal and are shown in Table I. A pretilt angle of $1^\circ$ and a wavelength of 550 nm are also used in the calculations. In both cases, $R = 100\%$ at $V = 0$ by definition. It can be seen that a threshold voltage of 1.5 V is obtained, followed by a decrease in reflectance. A minimum reflectance at about 2.5 V is obtained in both cases. At high voltages above 10 V, $R$ approaches 100% again. It corresponds to the LC cell becoming homeotropically aligned. For the HFE, a minimum of $R = 10\%$ is reached at 2.6 V. Hence the contrast of this display will be 10:1. For the optimized RTN, the RVC goes through a minimum reflection of nearly 0%. Hence the contrast will be extremely high for this particular wavelength. So for a NW display, it makes a big improvement to use the new optimized RTN.

### IV. OPTIMIZATION OF THE RTN DISPLAY

As mentioned above, the initial ($\phi, d\Delta n$) value of the RTN cell for optimization should be just above the first TN-ECB mode (of 63.6°, 0.19 $\mu m$). This corresponds to the combination (of 63.6°, 0.5 $\mu m$). Starting with these values, we varied ($\alpha, \phi, d\Delta n$) systematically. For each combination of $\phi, d\Delta n,$ and $\alpha$, the entire reflectance-voltage curve has to be calculated using a full dynamical solution. These RVCs have to be obtained at the three primary colors of red, green, and blue (RGB) in order to examine their color dispersion. The best combination will then be chosen as the optimum.

The choice of the optimal ($\alpha, \phi, d\Delta n$) combination has to be guided by the parameter space diagrams. In the following discussions, a NW display will be assumed. Our approach is (1) find a ($\alpha, \phi, d\Delta n$) combination that will produce a broad $R = 100\%$ region for different wavelengths at $V = 0$. (2) For each such ($\alpha, \phi, d\Delta n$) combination, we perform the calculation to obtain the dynamic RVCs to see which ones will produce the lowest reflectance and are the least dispersive when a voltage is applied.

For several values of $\phi$ near 63.6°, we also plot the $\alpha - d\Delta n$ parameter space. From each of these parameter space diagrams, we then can find the combinations of $d\Delta n$ and $\alpha$ that will give a reflectance of 100% and where the sensitivity to $\Delta n$ change is small. These points are actually quite easy to identify since they always correspond to a broadband peak. Figure 5 is an example of a $\alpha - d\Delta n$ parameter space for $\phi = 52^\circ$. The constant reflectance contours are in steps of 10%, starting with 100% along the $x$ axis.

### TABLE I. Parameters used in the numerical simulations.

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<th>Parameters</th>
<th>Value</th>
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<td>12.6e–12</td>
</tr>
<tr>
<td>$K_{22}$</td>
<td>6.1e–12</td>
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<tr>
<td>$K_{33}$</td>
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difference whether one chooses the HFE or the RTN mode of operation.

Figure 7 shows the RVCs for the RTN mode calculated at three wavelengths corresponding to red, green, and blue. There is a shift in the RVC between the three colors implying some degree of dispersion. However, it should be noted that the RVCs are more or less parallel below the minimum reflectance point. Thus in an active matrix controlled display, it should be possible to adjust the voltage of the subpixels belonging to the three primary colors to achieve excellent dark states and contrast for all colors. Excellent grayscale should also be possible using the same voltage compensation scheme. The complete reflectance versus wavelength curves are obtained for both the HFE and RTN modes. It is shown in Fig. 8 for both the field-off state (0 V) and the field-on state (2.6 V) for the NW situation. Within the visible region, both modes show a ~7% change in reflectance in both the dark and bright states.

V. EXPERIMENTAL RESULTS

To verify the theoretical simulations, sample cells were made using the various cell conditions discussed above. We used the MLC-5300 and 5400 multibottle systems from Merck in order to obtain a mixture with the desired $\Delta n$. Rubbed polyimide films were used to aligned the LC medium to produce the desired twist angles. One side of the LC cell has the usual indium–tin–oxide coating as the transparent electrode while the other surface was coated with aluminum to serve as the reflective electrode. A green laser at 514 nm was used for the RVC measurement while a Photo Research SpectroScan PR650 spectrometer was used for the spectral measurement.

Figure 9 shows the measured reflectance-voltage curves for the 45° HFE and the optimized RTN cells using a HeNe laser. A // polarizer geometry was used. The agreement between the curves in Figs. 6 and 9 is excellent. Even the threshold voltage measured is as predicted, despite the uncertainties in the elastic parameters used in the simulation due to mixing of different liquid crystals to adjust the $\Delta n$ value. As expected, the RTN cell gives near zero reflectance at 2.2 V whereas the HFE has a minimum reflectance of about 9%. Therefore the measured contrast ratio of the RTN display is well over 200:1 for this wavelength. Figure 10

FIG. 7. Calculated reflectance-voltage curves for the RTN mode at three wavelengths (RGB). Notice that the curves are parallel.

FIG. 8. Calculated reflectance spectra for the HFE and RTN modes at $V = 0$ (upper curves) and $V = 2.6$ V (lower curves).

FIG. 9. Measured reflectance-voltage curves for a RTN (squares) and a HFE (triangles) cell in NW operation. Note that the RTN minimum reflectance is near zero as predicted.

FIG. 10. Measured reflectance spectra for a RTN (squares) and a HFE (triangles) cell in NW operation.
shows the complete measured reflectance spectrum for this NW operation. Again the agreement with the theoretical curves in Fig. 8 is quite good.

Figures 11 and 12 show the results of a similar measurement using a NB \( \parallel \perp \) polarizer arrangement. For this measurement, instead of using a single polarizing beam splitter as shown in Fig. 3, we used a nonpolarizing 50/50 beam splitter and added two linear polarizers at the input and output arms. The two polarizers are cross polarized to simulate the situation of a polarizing beam splitter. The reason is that a single polarizing beam splitter cannot cover the entire visible range whereas a linear polarizer can. The linear polarizer (Melles Griot 03FP007) has almost no dispersion between 350 and 650 nm.

The experimental results shown in Figs. 11 and 12 are as expected as well. The minimum in Fig. 9 becomes a maximum in Fig. 11. The contrast ratio of both the HFE and RTN is now excellent since the reflectance at \( V = 0 \) is near zero in both cases. The measured reflectance spectra in Fig. 12 for both the field-off and field-on states for the HFE and RTN modes show very little wavelength dispersion. As a matter of fact, the variation in reflectance is approximately 5% within the visible spectrum. This is better than the NW case in Fig. 10 and is actually comparable to the case of the waveguiding Mauguin modes in ordinary TN displays.\(^{20}\) In this NB operation, there is not a big difference between the HFE and the RTN modes in terms of contrast and dispersion. The major difference is that the RTN is about 10% brighter.

From the theoretical and experimental results, we conclude that the RTN cell has high light efficiency, good chromatic dispersion, and high contrast ratio in both direct view (\( \parallel \perp \)) and projection (\( \parallel \perp \)) operations. The experimental contrast ratio is over 200:1 with narrow band input light. With white light input, the contrast is about 20:1.

VI. CONCLUSIONS

In summary, we have demonstrated a reflective LCD where the rear polarizer has been removed, without employing any retardation compensation.\(^4\) The use of a full parameter space approach has been applied to physically understand the operation of such displays and to guide the optimization process. We show that all reflective nematic type LCDs can be depicted on the parameter space, including the modes reported in the literature and the new RTN discussed here. We also show that the MTN and SCTN modes are based on optimizing the TN-ECB mode, and the RTN is based on optimizing the HFE mode. All these modes can be classified according to whether they fall inside or outside the TN-ECB wells. The two groups of reflective LCDs have quite different RVC characteristics.

We obtained the exact optimal operating conditions of the RTN using a full dynamic simulation of the LCD. Both the operating RVC and the reflectance spectrum of the RTN in both the NW and NB modes were calculated. These calculations were confirmed by experimental measurements on sample LCD cells. This display mode can work well with a parallel polarizer geometry or a perpendicular polarizer geometry. The former corresponds to a plane polarizer and is suitable for direct view situations. The latter geometry corresponds to a prism cube polarizer and is useful for projection displays.

The major advantages of the RTN mode are (1) a good dark state can be obtained. Hence the contrast ratio can be very good. (2) The wavelength dispersion is quite small. It is comparable to the normal Mauguin minimum TN modes.\(^{20}\) (3) The \( d\Delta n \) value is near 0.5 \( \mu m \) so that the cell gap can be reasonably large. This is important in making practical cells. (4) The light efficiency is high. Since the output light is also linearly polarized, the input light is utilized fully, unlike the cases of HFE and 90° MTN. (5) Since the homeotropic state is not involved, the operating voltage is generally lower than the “in-well” modes.

The RTN has a reflectance-voltage curve that is similar to ordinary TN displays. Hence they are not too appropriate for multiplex driving. The are suitable, however, in active matrix displays where the rear polarizer has been eliminated or is impossible to put in, as in the case of photoaddressed LCLV or silicon backplane projectors. The RTN can also work well with active matrix backplanes in a direct view geometry. In that case, a high contrast display should be possible without the use of a retardation film which is needed for the MTN mode.\(^{19}\) Moreover, the cell gap of about 5 \( \mu m \)

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**FIG. 11.** Measured reflectance-voltage curves for a RTN (squares) and a HFE (triangles) cell in NB operation.

**FIG. 12.** Measured reflectance spectra for a RTN (squares) and a HFE (triangles) cell in NB operation.
is much easier to work with than the less than 2 \( \mu \text{m} \) cell gap. It is believed that the RTN will find useful applications in many practical systems in both projection and direct view situations.

**ACKNOWLEDGMENT**

This research was supported by the Hong Kong Industry Department.

23. Notice that we have presented the Jones matrix slightly differently from that in Ref. 18. Here the sign of \( B \) is reversed. This sign has to do with whether the cell has a right- or left-handed twist. Here we assume that the twist angle of the LC cell is \( -\phi \). As long as the signs are consistent, it does not matter at the end of the calculation.