

Mueller calculus and perfect polarization conversion modes in liquid crystal displays

S. T. Tang and H. S. Kwok^{a)}

*Centre for Display Research, Department of Electrical and Electronic Engineering,
Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong*

(Received 10 October 2000; accepted for publication 21 February 2001)

We introduce a 4×4 Mueller matrix for describing the polarization states of a liquid crystal cell. Using this Mueller matrix, it is possible to derive conditions whereby a linearly polarized input can be converted to either a perfectly linearly or perfectly circularly polarized output. These are called the perfect polarization conversion (PPC) conditions. These PPC conditions can be used in many different ways. For example, based on these PPC conditions, polarization mode switching schemes can be derived for the analysis of most liquid crystal displays. © 2001 American Institute of Physics. [DOI: 10.1063/1.1365443]

I. INTRODUCTION

Liquid crystal displays (LCD) are used widely in many products. Most of the LCDs operate in the so-called waveguiding 90° twisted nematic (TN) mode. Additionally, the supertwisted nematic (STN) mode LCD with a twist angle of 180° – 240° is used in many passively addressed displays.¹ Some products also employ the “high twisted” TN (HTN) mode with a twist angle of 100° – 150° . The analysis of these LCD modes has been piecemeal in the sense that each individual display was analyzed separately. There was no overall picture of the design of various LCD modes, and the relationship between these modes is not clearly known.

We have previously presented a unified picture of all homogeneously twisted LCDs in terms of a parameter space diagram.^{2,3} The idea is that the voltage-off optical properties of any liquid crystal display are determined mostly by its twist angle ϕ and retardation $d\Delta n$. (The retardation can also be represented by an angle δ which is equal to $\pi d\Delta n/\lambda$ where λ is the wavelength of light.) Together with the input polarizer angle α , and the output analyzer angle γ , all the important optical properties such as the transmission and reflectance spectra, the contrast ratio, can be calculated.

The parameter space diagram approach has been quite useful in visualizing the relationship between various LCD modes. It was developed using the 2×2 Jones matrix. It is also useful in the design of new LCD modes such as the reflective, single polarizer LCD.^{3–5} In this article, we carry the concept of this parameter space one step further by introducing the idea of perfect polarization conversion (PPC) modes. In order to derive these PPC modes, we use the 4×4 Mueller matrix description of the LCD cell. The polarization optics of these LCDs can be systematically presented in this picture. These PPC modes are very useful in understanding the operation and optical properties of the LCD. Moreover, it depicts clearly the working relationship between various LCD modes. We shall apply these newly for-

mulated PPC modes to analyze a few new types of LCDs such as the bistable twisted nematic LCD and the single polarizer reflective LCD. This treatment of the LCD optical modes by means of their specific polarization properties can be termed the polarization mode-switching scheme.

II. THEORY

In general, a LC cell will turn a linearly polarized input light into an elliptically polarized light. As a matter of fact, it is well-known that a nematic LC cell behaves optically as a combination of a polarization rotator and a retardation plate.⁶ The objective of this article is to find the conditions for the LC cell, namely the values of (α, ϕ, δ) such that the output light is either perfectly linearly polarized or perfectly circularly polarized. These are called the perfect polarization conversion (PPC) conditions. It should be noted that all angles, α , γ , and ϕ are referenced to the input director of the LC cell.

The simplest approach is to use the Jones matrix description of the LC cell. It has been shown that the Jones matrix of a LC cell is given by the following:^{3–5}

$$M_{LC} = \begin{pmatrix} a - ib & -c - id \\ c - id & a + ib \end{pmatrix}, \quad (1)$$

where

$$a = \cos \phi \cos \chi + \frac{\phi}{\chi} \sin \phi \sin \chi,$$

$$b = \frac{\delta}{\chi} \cos \phi \sin \chi,$$

$$c = \sin \phi \cos \chi - \frac{\phi}{\chi} \cos \phi \sin \chi,$$

$$d = \frac{\delta}{\chi} \sin \phi \sin \chi,$$

and

$$\chi^2 = \phi^2 + \delta^2.$$

^{a)}Electronic mail: eekwok@usthk.ust.hk

If the polarization of the input light is at an angle α to the input director, then the output polarization is given by

$$\begin{pmatrix} x \\ y \end{pmatrix} = M_{LC} \begin{pmatrix} \cos \alpha \\ \sin \alpha \end{pmatrix}. \tag{2}$$

Therefore, in order to have a linearly polarized output, we can simply require that x and y be real. Likewise, for a circularly polarized output, we can require that the phases of x and y differ by $\pm 90^\circ$ and amplitudes equal.

An alternative approach is possible and is more intuitive, and is the subject of this discussion. It relies on the formulation of the polarization optics in terms of the Stokes vector and the Mueller matrix.⁷ It is more elegant, as any polarization state can be represented as a point on the Poincare sphere. The Mueller matrix representation of a general twisted liquid crystal cell U_M can be obtained from its Jones matrix M_{LC} by the following formula:⁸

$$U_M = T(M_{LC} \otimes M_{LC}^*) T^{-1}, \tag{3}$$

where

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & i & -i & 0 \end{pmatrix}, \tag{4}$$

and the direct matrix product of a 2×2 matrix U is defined as

$$U \otimes U^* = \begin{pmatrix} u_{11}u_{11}^* & u_{11}u_{12}^* & u_{12}u_{11}^* & u_{12}u_{12}^* \\ u_{11}u_{21}^* & u_{11}u_{22}^* & u_{12}u_{21}^* & u_{12}u_{22}^* \\ u_{21}u_{11}^* & u_{21}u_{12}^* & u_{22}u_{11}^* & u_{22}u_{12}^* \\ u_{21}u_{21}^* & u_{21}u_{22}^* & u_{22}u_{21}^* & u_{22}u_{22}^* \end{pmatrix}. \tag{5}$$

After some algebra, it can be shown that the resultant Mueller matrix is given by

$$U_M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-2(c^2+d^2) & 2(bd-ac) & -2(ad+bc) \\ 0 & 2(ac+bd) & 1-2(b^2+c^2) & 2(ab-cd) \\ 0 & 2(ad-bc) & -2(ab+cd) & 1-2(b^2+d^2) \end{pmatrix}. \tag{6}$$

Alternatively, we can write U_M as

$$U_M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & A & B & C \\ 0 & D & E & F \\ 0 & G & H & K \end{pmatrix}. \tag{7}$$

It should be recalled that in the Mueller matrix formulation, the polarization state of any light is given by the Stokes vector. Thus for any light that is polarized at α to the input director, the resultant Stokes vector S' of the light after passing through the LC cell is given by

$$S' = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & A & B & C \\ 0 & D & E & F \\ 0 & G & H & K \end{pmatrix} \begin{pmatrix} 1 \\ \cos 2\alpha \\ \sin 2\alpha \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ A \cos 2\alpha + B \sin 2\alpha \\ D \cos 2\alpha + E \sin 2\alpha \\ G \cos 2\alpha + H \sin 2\alpha \end{pmatrix}. \tag{8}$$

Note that the Stokes vector as given in Eq. (8) has the property that $S_1^2 + S_2^2 + S_3^2 = 1$. In fact, (S_1, S_2, S_3) can be regarded as the Cartesian coordinates of the Stokes vector on the Poincare sphere.⁷ Equation (8) can be used to find all the PPC modes of the LC cell.

III. LINEAR PPC MODES

A linearly polarized output will require that the S_3 component of the Stokes vector to be zero. Graphically, it is the same as having the Stokes vector to lie on the equator of the Poincare sphere. Thus, we have

$$G \cos 2\alpha + H \sin 2\alpha = 0. \tag{9}$$

After some algebra, it can be shown that this condition can be written as

$$\frac{\delta}{\chi} \sin \chi \left(\frac{\phi}{\chi} \sin \chi \cos 2\alpha - \cos \chi \sin 2\alpha \right) = 0. \tag{10}$$

Equation (10) can be solved more easily than requiring x and y to be real in Eq. (2). $\delta=0$ is a trivial solution which can be ignored. There are two nontrivial solutions of this equation.

A. Solution I: $\sin \chi = 0$

$\sin \chi = 0$ is an obvious solution of Eq. (10). Thus, we have

$$\chi = N\pi \quad \text{where } N=0,1,2,3,\dots \tag{11}$$

It implies that

$$\left(\frac{d\Delta n}{\lambda} \right)^2 + \left(\frac{\phi}{\pi} \right)^2 = N^2. \tag{12}$$

These are called the LP1 solutions. Equation (12) is the parametric equation of a circle in the parameter space (ϕ, δ) . Figure 1 is a plot showing the first three orders ($N=1,2,3$). $N=0$ gives a trivial solution and is ignored. Further simplifying Eq. (12) gives the required retardation of the LC cell for these LP1 solutions as

$$d\Delta n = \lambda \sqrt{N^2 - (\phi/\pi)^2}. \tag{13}$$

Under the LP1 condition, the Mueller matrix of the LC cell reduces to

$$U_M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\phi & -\sin 2\phi & 0 \\ 0 & \sin 2\phi & \cos 2\phi & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{14}$$

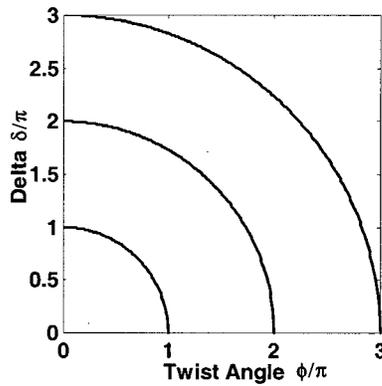


FIG. 1. LP1 solutions plotted on the (ϕ, δ) parameter space. Different circles represent different orders of the solution. The smallest circle is $N = 1$, etc.

which is a polarization rotator matrix with angle ϕ . Thus the LC cell behaves as a pure polarization rotator. The output is linearly polarized at an angle $\phi + \alpha$ to the x axis. This situation is actually a generalization of the waveguiding condition for the TN cell with $\alpha = 0^\circ$ or 90° . The various values of N correspond to the various Mauguin minima.⁹ The LP1 solution is actually a generalization of the time-honored waveguiding modes.

B. Solution II: $\sin \chi \neq 0$

For $\sin \chi \neq 0$, Eq. (10) requires

$$\frac{\phi}{\chi} \sin \chi \cos 2\alpha - \cos \chi \sin 2\alpha = 0. \tag{15}$$

Rearranging Eq. (15) leads to Eq. (16):

$$\tan 2\alpha = \frac{\phi}{\chi} \tan \chi. \tag{16}$$

Substituting Eq. (16) into (8) gives

$$S' = \begin{pmatrix} 1 \\ \cos 2(\phi - \alpha) \\ \sin 2(\phi - \alpha) \\ 0 \end{pmatrix}. \tag{17}$$

Thus the output is linearly polarized at an angle of $\phi - \alpha$ to the x axis. Equation (16) defines the LP2 solution. Under this condition, the linearly polarized input is converted to a linearly polarized output at an angle $\phi - \alpha$. One important difference between the LP1 and LP2 solutions is that the retardation value of the LC cell is dependent on the input polarizer angle α . Figure 2 is a plot of Eq. (16) with different input polarizer angle α . Many curves are obtained for various values of α . It is obvious that the LP2 solutions span the whole (ϕ, δ) parameter space. An important corollary is that for any (ϕ, δ) , there exists an α such that Eq. (16) is satisfied. That is, a LP2 mode always exists for any LC cell. This is an important result with great consequences.

It should be noted that if $\alpha = 0^\circ$ or 45° , the solution for the LP2 mode corresponds to a set of circles in the (ϕ, δ) parameter space, similar to the LP1 solutions. Actually, for the case of $\alpha = 0^\circ$ or 90° , the LP1 and LP2 solutions are

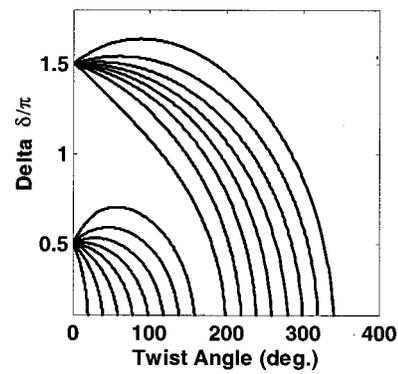


FIG. 2. LP2 solutions plotted on the (ϕ, δ) parameter space. The curves shown are the first two N values with different input polarizer angle α . The input polarizer angle increases from 10° to 80° from left- to right-hand side.

degenerate. The circles corresponding to the LP1 and LP2 modes with $\alpha = 45^\circ$ interleave each other in the (ϕ, δ) parameter space.

To summarize the results, there are in general two sets of linear polarization preservation solutions. They are

LP1 $d\Delta n = \lambda \sqrt{N^2 - (\phi/\pi)^2}, \tag{18}$

$\gamma = \phi + \alpha \pm m\pi, \tag{19}$

LP2 $\tan 2\alpha = \frac{\phi}{\chi} \tan \chi, \tag{20}$

$\gamma = \phi - \alpha \pm m\pi, \tag{21}$

where $N = 1, 2, \dots$ and $m = 0, 1, 2, \dots$

IV. CIRCULARLY POLARIZED OUTPUT

In order to have a circularly polarized output, we need the output Stokes vector to lie either on the North Pole or South Pole of the Poincare sphere. Mathematically, this condition is the same as requiring the magnitude of S_3 in the Stokes vector to be unity in Eq. (8). Alternatively, it is equivalent to requiring $S_1 = S_2 = 0$. Using the latter condition, we derive the following simple condition for circular polarization (CP) output:

$AE = BD. \tag{22}$

This can be simplified to the following CP condition:

$$\frac{\delta}{\chi} \sin \chi = \pm \frac{1}{\sqrt{2}}. \tag{23}$$

For each solution (ϕ, δ) obtained from Eq. (23), the corresponding polarizer angle is given by

$$\tan 2\alpha = -\frac{D}{E} = -\frac{\chi}{\phi} \cot \chi. \tag{24}$$

The physical meaning of Eqs. (23) and (24) is that for each polarizer angle α , there is a unique set of (ϕ, δ) values such that perfect circular polarized output is obtained. As α is varied, the set of solutions (ϕ, δ) will trace out a curve in the parameter space as given by Eq. (23). Note that the situation is different from the LP2 solution case, where a continuous

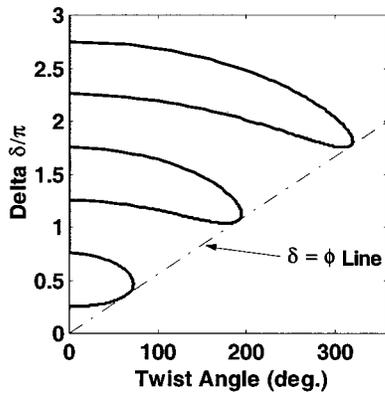


FIG. 3. CP solutions plotted on the (ϕ, δ) parameter space. No solution can be obtained for $\phi > \delta$.

curve of (ϕ, δ) exists for each given α . Here, because of conditions (23) and (24) simultaneously, only a finite set of (ϕ, δ) solutions exists for each α .

The CP solutions are shown in the (ϕ, δ) parameter space in Fig. 3. Each branch in Fig. 3 corresponds to a value of N in the solution of Eq. (23). Note that in the highly twisted region where $\phi > \delta$, there is no CP solution. Note also that if $\alpha = 0$, the solution of Eq. (23) gives $\delta = \phi$. This is simply the TN-ECB solution obtained by Sonehara *et al.*¹⁰ In Fig. 3, it can be seen that the $\phi = \delta$ line touches the tip of the CP solutions in the (ϕ, δ) parameter space. Figure 4 is a plot showing the corresponding α values for different CP solutions. Each curve corresponds to a higher order CP mode.

It is also interesting to note that Eq. (24) is remarkably similar to Eq. (20). The two equations are the same if $\tan 2\alpha_{CP} = -\cot 2\alpha_{LP2}$. This means that it is possible to obtain a LP2 and a CP solution for the same (ϕ, δ) values but their input polarizer angles must differ exactly by $\pi/4$.

Thus we have derived all the PPC conditions of a LC cell. They are given by Eqs. (18), (20), (23), and (24).

V. APPLICATIONS

In the last few sections, we have derived the three perfect polarization conversion modes for a twisted nematic cell. They are the LP1, LP2, and CP modes. For a linearly polarized light at angle α to the input director, there are

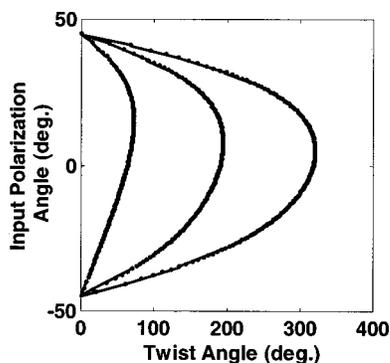


FIG. 4. Input polarizer angle as a function of the twist angle for the CP mode. The three curves here represent $N = 1, 2, 3$.

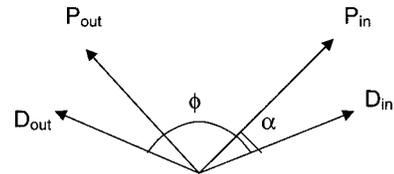


FIG. 5. Polarizer and director angles for the self-compensating geometry.

certain conditions of (ϕ, δ) such that the output will be linearly polarized with an angle $\phi + \alpha$ to the input director (LP1), or linearly polarized at an angle $\phi - \alpha$ to the input director (LP2), or circularly polarized (CP). The set of (ϕ, δ) values form a curve in the (ϕ, δ) parameter space and are shown in Figs. 1, 2, and 3. We can make use of these results to analyze a few typical situations.

A. TN modes

Normal LCDs operate in the waveguiding TN modes. In the Gooch and Tarry analysis of the Mauguin modes,⁹ the following formulas were derived for the transmission coefficient of a LC cell with the input polarizer parallel to the input director and the output analyzer cross to the output director:

$$T = \frac{1}{1 + u^2} \sin^2 \sqrt{1 + u^2}, \tag{25}$$

where

$$u = \frac{\delta}{\phi} = \frac{\pi d \Delta n}{\lambda \phi}.$$

In particular, for a 90° TN cell, the Mauguin minima are given by the zeros of Eq. (25) as

$$\frac{d \Delta n}{\lambda} = \sqrt{N^2 - \frac{1}{4}}. \tag{26}$$

This is exactly the same result as given by the LP1 modes in Eq. (18), if we let $\phi = \pi/2$. Thus the TN waveguiding modes can be regarded as LCD operating between the LP1 mode and the homeotropic (H) mode where all the LC molecules are perpendicular to the LC cell. Notice that the waveguiding mode requires α to be 0° or 90° . For the LP1 mode, α can be any value. It is therefore correct in saying that the LP1 modes are generalization of the Mauguin waveguiding modes.

B. Self-compensated modes

The self-compensated modes can be regarded as LCD operating between the LP2 mode and the H mode. In the self-compensation geometry, the input–output directors and the input–output polarizers are placed symmetrically as shown in Fig. 5. The advantage of such geometry is that the LC director deformation is symmetric, with the birefringence compensating each other in the horizontal direction and the best viewing direction is along the vertical axis.¹¹ The operating voltage for such a LCD mode is lower than any other modes. The high twist or HTN modes are in this geometry. For this HTN, the nonselect state is the LP2 twist state with

TABLE I. Classification of common transmissive general TN displays according to their switching modes. These are all normally white modes. Retardation values are calculated assuming $\lambda = 550$ nm.

Switching modes	Optical modes	α	γ	$(\phi, d\Delta n)$
LP1→H	TN	0°	90°	(90°, 0.48 μm), (90°, 1.07 μm)
LP2→H	LTN, HTN	$(\phi - 90^\circ)/2$	$(\phi + 90^\circ)/2$	(30°, 0.29 μm), (120°, 0.59 μm)
LP1→LP2*	STN	45°	$\phi + 45^\circ$	(240°, 0.82 μm)

$\gamma_{LP2} = \phi - \alpha$. The select state is the homeotropic state with $\gamma_H = \alpha$. Hence, in order for the select and nonselect states to be orthogonal, it is required that

$$\alpha = (\phi - 90^\circ)/2. \tag{27}$$

Row 2 in Table I gives examples of the LC cell retardation needed for a good optical efficiency 120° HTN mode. In addition to the HTN displays, the low twist TN (LTN) modes also belong to this category. It is noted that for these self-compensated LP2 modes, the retardation value required increases as the twist angle increases (see Fig. 8 below).

C. Supertwisted nematic mode

STN LCDs are used extensively in displays requiring higher information content than HTN and TN LCD. The optimization of the STN has been extensively discussed. We shall attempt to analyze the STN LCD using the LP modes described in this article. For the STN LCD, both the voltage-off and voltage-on states should be LP modes. It is generally true that the voltage-on (nonselect) state is a LP1 state. Therefore for a 240° STN cell, the retardation is determined to be around 0.82 μm from Eq. (18). In order to have an optically optimized STN, it is necessary that the output polarization of the voltage-on (select) state to be a LP mode as well. It is important to realize that with a nonuniformly twisted cell, the PPC modes as discussed in the prior section are no longer valid. The voltage-on state is not the H state due to the large residual twist and tilt. It has been shown in this intermediate voltage state that there always exists a pair of orthogonal linear polarization directions such that the output polarization is also linearly polarized.¹² Following Aben, these directions are called the *primary characteristic directions*. Thus we have to require α to coincide with one of these characteristics directions of the LC layer under an applied voltage. The special symmetry properties of the LC molecules under any voltage is

$$(1) \text{ Twist angle } \phi(z) - \phi(d-z) = \phi_T, \tag{28}$$

$$(2) \text{ Tilt angle } \theta(z) = \theta(d-z), \tag{29}$$

where ϕ_T is the total twist of the cell, $\phi(z)$ and $\theta(z)$ are the twist and tilt as a function of the z position, d is the cell thickness. At any voltage, there will be a condition of the LC layer such that the output is linearly polarized at $(\phi - \alpha)$ to the input director, similar to the LP2 mode. This condition can be called the LP2* mode. This LP2* condition and the ordinary LP2 condition are different since Eq. (20) applies

only to the uniform twist case. In order to have maximum contrast, the output polarizer direction γ_p is set orthogonal to $(\phi - \alpha)$. Thus, we have

$$\gamma_p = \phi - \alpha \pm 90^\circ. \tag{30}$$

For maximum brightness, the output polarizer direction γ_p is set parallel to $(\phi + \alpha)$, the output polarization of the LP1 mode. Thus, we have

$$\gamma_p = \phi + \alpha. \tag{31}$$

For both maximum brightness and contrast to occur, Eq. (30) must equal (31), thus,

$$\alpha = \pm 45^\circ. \tag{32}$$

Equation (32) implies that in order to have optically optimized STN display, we must have the primary characteristic direction at $\pm 45^\circ$ to the input director in the activated state. Then the input and output polarizers are oriented at $\alpha = \pm 45^\circ$ and $\gamma = \phi \pm 45^\circ$, respectively. The blue and yellow mode of the STN LCD depends on whether one chooses the LP1 mode to be the bright state or the dark state, respectively.

The primary characteristic direction depends on the particular configuration of the LC layer, which in turn depends on the applied voltage for a particular LC cell. There is no guarantee that a primary characteristic direction of $\pm 45^\circ$ should exist by simply considering the cell parameters such as twist angle and retardation. A complete simulation with exact physical parameters such as elastic constants, dielectric constants, helical pitch, and pretilt angle is required. Figure 6 presents results of the calculated primary characteristic direc-

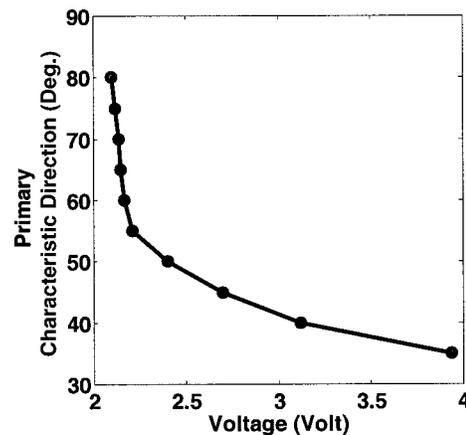


FIG. 6. Primary characteristic direction as a function of the applied voltage for a 240° STN cell.

tion as a function of the applied voltage for a 240° STN. The LC cell parameters are $K_1=12.5$ pN, $K_2=7.3$ pN, $K_3=17.9$ pN, $\epsilon_{\parallel}=14.1$, $\epsilon_{\perp}=4.1$, $n_0=1.502$, $n_e=1.638$, pitch = 10 μm , $d=6.05$ μm , pretilt angle = 5°. These are values for typical STN LCD. In general, it is our experience that for 240° STN cells, the 45° primary characteristic direction can always be obtained at moderate voltages. However, for higher twist angles, such as 270°, the 45° characteristic direction is not possible.

Hence an ‘‘ideal’’ STN display can be regarded as a display that switches between the LP1 and LP2* modes for the nonselect and select states. Notice that the above discussion totally ignores the dispersion properties of the display. Most STNs including the yellow and blue modes obey this condition. However, there are STN displays that sacrifice this optically optimized design for other considerations such as a better multiplexibility.

The following general statements can be made for STN optical mode design: (1) The (ϕ, δ) values are determined by the voltage-off state LP1. Therefore the retardation decreases as the twist angle increases. (2) α should be at $\pm 45^\circ$ to obtain unity output transmittance and maximum contrast. (3) The polarizer angles are determined by the LP2* relations. Specifically, the rear polarizer should be perpendicular to the LP2* output direction in order to have a completely dark select state.

D. Bistable TN LCD modes

The optics of bistable twisted nematic (BTN) displays have been extensively investigated by us as well as others.^{13–17} Both the 2×2 and the 4×4 matrix approach can be used. BTN displays have two stable twist states which differ by an angle of 2π at 0 V. Therefore, one twist state must be a LP1 mode and the other twist state should be a LP2 mode. Moreover, the output polarization of these two twist states must be orthogonal to each other in order for the display to have a peak transmittance of unity and a minimum transmittance of zero, at one wavelength. Thus Eq. (32) is also true for the BTN, which implies that α must be $\pm 45^\circ$ as in the case of STN displays as well. This is an interesting result that has not been anticipated by all previous studies of BTN.

For an input polarized light angle of 45°, the LP2 condition of Eq. (16) reduces to

$$\delta^2 + \phi^2 = (M - 0.5)^2 \pi^2. \tag{33}$$

Further requiring the LP1 and LP2 modes to have the same retardation (which is obvious since the same LC cell is used) and solving Eqs. (12) and (33) simultaneously, one can obtain all the BTN modes that have a peak transmittance of unity and a low transmittance of zero. The results are listed in Table II. The relationship between the LP1 and LP2 states for the BTN is illustrated in Fig. 7 graphically. The LP1 and LP2 modes are related by a horizontal line with length 2π . It should be noted that the best operating mode for a transmissive BTN has twist angles of -11° and 349° . All previous studies of BTN use the twist angles of 0 and 2π . That choice does not correspond to the best possible optical properties.

TABLE II. First ten transmissive BTN solutions. The sign of the two metastable twist states ϕ_1 and ϕ_2 can be reversed while representing the same mode. α is always 45° and γ is set for maximum contrast operation.

$d\Delta n$ (μm)	ϕ_1	ϕ_2	γ
0.273	-11	349	124
0.3995	-124	236	-79
0.523	56	416	101
0.733	124	484	259
0.765	-101	259	34
0.932	191	551	236
1.0735	-79	281	-34
1.125	259	619	394
1.364	-57	303	78
1.647	-33	327	12

This is a good example to show the powerful technique of the polarization conversion modes analysis as applied to the case of BTN.

E. Reflective single polarizer LCD modes

Single polarizer reflective LCD has been studied because of their many advantages such as the elimination of a viewing shadow and increased brightness. We, as well as many others have studied this display.^{3–5,18–20} Many modes, such as the TN-ECB, hybrid-field effect (HFE), reflective-TN (RTN), mixed-mode TN (MTN), self-compensated TN (SCTN), and mixed-TN-and-ECB (MTB) modes have been discussed. Using the present formulation, a reflective LCD should have one of its states being a CP mode, and the other state should either be the H mode or LP mode. Two polarization switching types are therefore possible. They are the CP to homeotropic switching and the LP1 to CP* switching. The CP* mode is defined as a CP-like mode for the LCD under an applied voltage. With the applied voltage, Eqs. (23) and (24) are not obeyed, but the polarization condition is satisfied, i.e., a circularly polarized output is obtained. The situation is similar to the LP2* intermediate voltage state in the STN display.

Representative modes with their design parameters are listed in Table III. It is easy to verify that if the LC cell is in

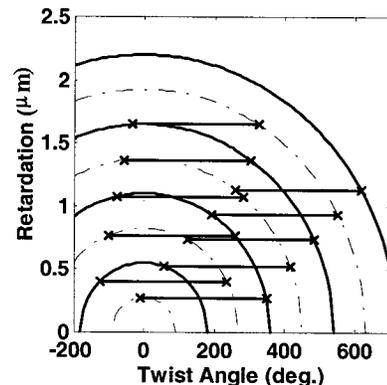


FIG. 7. Graphical solutions for obtaining the BTN modes. The LP1 (solid) and LP2 (dashed) modes are plotted as circles. A solution exists whenever a horizontal line of length 2π can link a LP1 and a LP2 state.

TABLE III. Classification of reflective TN displays according to their switching modes.

Switching modes	Optical modes	α	$(\phi, d\Delta n)$	Normal modes
CP→H	TN-ECB	0°	(63.6°, 0.194 μm) (190.9°, 0.583 μm)	NB
CP→H	MTN	20°	(90°, 0.25 μm)	NB
CP→H	SCTN	30°	(60°, 0.35 μm)	NB
LP1→CP*	HFE	0°	(45°, 0.54 μm)	NW
LP1→CP*	RTN	0°	(52°, 0.52 μm)	NW
LP1→CP*	RSTN	6°	(240°, 0.85 μm)	NW

the H or LP mode, the output will be a bright state, if a linear polarizer is used as both the input and output polarizer. Also if the LC is in the CP state, the output will be the dark state. The CP to homeotropic switching mode is therefore a normally black (NB) mode while the $LP1$ to CP^* switching mode is a normally white (NW) mode. Of course, the NB and NW situations will be inverted if a polarizing beamsplitter is used, and the output is taken from the reflection direction. This is usually the case for reflective LCD used in projection displays. This situation has been analyzed quite thoroughly in previous publications.³⁻⁵

It should be noted that the values of the parameters given in Table III results from maximum contrast and brightness considerations alone. For actual devices, extensive simulations near these operating points are needed to optimize other properties such as dispersion, operating voltages, etc.³

VI. SUMMARY

We have introduced a new formulation for the analysis of polarization states in a LCD. In particular, we introduced the perfect polarization conversion (PPC) modes whereby the linearly polarized input results in a linearly polarized or circularly polarized output. Specific conditions for these PPC modes to occur have been derived. The analysis of these PPC modes is very useful in LCD optical mode design. It is found that all the practical LCD modes in the voltage-off state belong to one of these PPC solutions.

A polarization mode-switching scheme is used to explain the principle of operation of all the LCD, both in the voltage-on and-off states. We argue that most of the LCDs can be viewed as switching between the LP , CP , and H states. The latter state is the homeotropic state at high voltage. The PPC mode analysis is a very powerful method for LCD optical mode design. A schematic of all the major LCD modes in the (ϕ, δ) space is shown in Fig. 8.

It is believed that the PPC modes discussed here are very useful not only in understanding the operation of existing LCDs. It should also be a powerful tool in designing and optimizing new displays. Moreover, there are other applica-

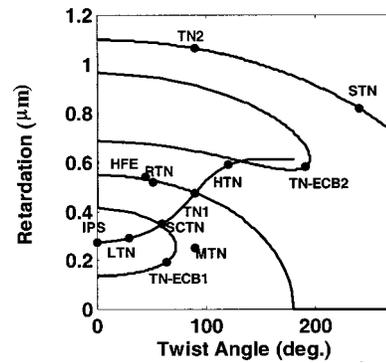


FIG. 8. All common LCD optical modes are depicted on the (ϕ, δ) parameter space. They all lie on one of the PPC solution curves. The slanted curve is the $LP2$ solution with continuously varying α .

tions of the 4×4 description of LCD modes. We have recently applied the PPC modes to the measurement of LCD twist angle and retardation values.^{21,22}

ACKNOWLEDGMENT

This research was supported by the Hong Kong Innovation and Technology Fund.

- ¹ V. G. Chigrinov, *Liquid Crystal Devices* (Artech House, Boston, 1999).
- ² H. S. Kwok, *J. Appl. Phys.* **80**, 3687 (1996).
- ³ H. S. Kwok, J. Chen, F. H. Yu, and H. C. Huang, *J. Soc. Inf. Disp.* **7**, 127 (1999).
- ⁴ S. T. Tang, F. H. Yu, J. Chen, M. Wong, H. C. Huang, and H. S. Kwok, *J. Appl. Phys.* **81**, 5924 (1997).
- ⁵ F. H. Yu, J. Chen, S. T. Tang, and H. S. Kwok, *J. Appl. Phys.* **82**, 5287 (1997).
- ⁶ H. Hurwitz and R. C. Jones, *J. Opt. Soc. Am.* **31**, 493 (1941).
- ⁷ M. Born and E. Wolf, *Principles of Optics* (Pergamon, Oxford, 1975).
- ⁸ *Physics of Liquid Crystalline Materials*, edited by I. C. Khoo and F. Simoni (Gordon and Breach Science, London, 1991), p. 337.
- ⁹ C. H. Gooch and H. A. Tarry, *J. Phys. D* **8**, 1575 (1975).
- ¹⁰ T. Sonehara and O. Okumura, *Japan Display, Proceedings of the Ninth International Display Research Conference, 1989*, pp. 192-195.
- ¹¹ K. H. Yang, *International Display Research Conference, Eurodisplay '96, 1996*, p. 449.
- ¹² H. Aben, *J. Opt. Soc. Am. A* **3**, 1414 (1986).
- ¹³ Z. L. Xie and H. S. Kwok, *J. Appl. Phys.* **84**, 77 (1998).
- ¹⁴ Z. L. Xie, H. J. Gao, S. Y. Xu, and H. S. Kwok, *J. Appl. Phys.* **86**, 2373 (1999).
- ¹⁵ Z. Z. Zhuang, Y. J. Kim, and J. S. Patel, *Appl. Phys. Lett.* **75**, 3008 (1999).
- ¹⁶ Z. L. Xie, M. Dong, S. Y. Xu, H. J. Gao, and H. S. Kwok, *J. Appl. Phys.* **87**, 2673 (2000).
- ¹⁷ S. T. Tang, H. W. Chiu, and H. S. Kwok, *J. Appl. Phys.* **87**, 632 (2000).
- ¹⁸ C. L. Kuo, C. K. Wei, S. T. Wu, and C. S. Wu, *Jpn. J. Appl. Phys., Part 1* **36**, 1077 (1997).
- ¹⁹ Y. Saitoh, Y. Yoshida, and H. Kamiya, *J. Soc. Inf. Disp.* **28**, 651 (1997).
- ²⁰ M. H. Schuck, D. J. McKnight, and K. M. Johnson, *J. Soc. Inf. Disp.* **27**, 403 (1996).
- ²¹ S. T. Tang and H. S. Kwok, *J. Soc. Inf. Disp.* **29**, 552 (1998).
- ²² S. T. Tang and H. S. Kwok, *J. Appl. Phys.* **89**, 80 (2001).